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Scheme dependence of NLO corrections to exclusive processes

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Abstract

We apply the so-called conformal subtraction scheme to predict perturbatively exclusive processes beyond leading order. Taking into account evolution effects, we study the scheme dependence for the photon-to-pion transition form factor and the electromagnetic pion form factor at next-to-leading order for different pion distribution amplitudes. Relying on the conformally covariant operator product expansion and using the known higher order results for polarized deep inelastic scattering, we are able to predict perturbative corrections to the hard-scattering amplitude of the photon-to-pion transition form factor beyond next-to-leading order in the conformal scheme restricted to the conformal limit of the theory.

Keywords:

exclusive processes, conformal symmetry, perturbative corrections, scheme dependence

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I. INTRODUCTION

An important theoretical issue of the perturbative QCD approach to exclusive processes is to check its self-consistency by studying perturbative corrections. This task was carried out in the modified minimal subtraction $\overline{\mathrm{MS}}$ scheme for the next-to-leading (NLO) corrections to the photon-to-pion transition form factor Ref. [1–3], the electromagnetic pion form factor [4–10], and the two-photon process $\gamma\gamma \to M^+M^-(M = \pi, K)$ [11]. The latter was only calculated for the special case of an equal momentum sharing distribution amplitude (DA). Only in a recent analysis of the electromagnetic pion form factor [10] has the NLO correction to the evolution of the DA been completely taken into account. Discrepancies in the one-loop approximation of the hard-scattering amplitude for the pion form factor were clarified in Refs. [8,9]. The NLO corrections to the pion form factor and to the process $\gamma\gamma \to M^+M^-$ are rather large at accessible momentum transfer; however, it turned out that for the equal momentum sharing DA the perturbative corrections to the integrated cross section of the charged meson production can almost be absorbed into the electromagnetic meson form factor (see Ref. [11]).

It was mentioned long ago that conformal symmetry plays an important role for the perturbative description of exclusive processes [12,13]. Unfortunately, conformal symmetry is broken by the renormalization of the UV-divergencies appearing in the hard-scattering part as well as in the DA. There are two different sources for the breaking of conformal symmetry induced by: (i) the running coupling and (ii) the renormalization of the DA. While the first one is proportional to the β-function and vanishes for a non-perturbative hypothetical fixed point, the latter appears in the minimal subtraction (MS) scheme; fortunately, it is absent in a special subtraction scheme, which we will call conformal subtraction (CS) scheme [14]. Let us point out that the Brodsky-Lepage-Mackenzie (BLM) scale setting prescription is suitable to absorb conformal anomalies, which are in the CS scheme only proportional to β, in the scale setting prescription of the coupling.

The understanding of conformal symmetry and its breaking in perturbative QCD has been applied recently for the NLO calculation of the leading twist-2 Wilson-coefficients [15] and the flavor singlet evolution kernels [16,17] relevant for off-forward processes as well as to prove the hypotheses of naive non-Abelianzation for the renormalon chain appearing in the evolution kernel [15]. In this paper we pursue the phenomenological consequences of the CS scheme for the exclusive processes mentioned above.

The rest of the paper is organized as follow. In Section II we discuss the scheme dependence of the perturbative corrections and argue that the NLO corrections calculated in the $\overline{\mathrm{MS}}$ scheme should be reanalyzed in this special scheme that ensures conformal symmetry. The consequences of the BLM scale fixing prescription and the evolution effects of the pseudo-scalar meson DA to NLO are analyzed in the flavor nonsinglet sector in Section III. In Sections IV and V we numerically investigate the NLO corrections for the photon-to-pion transition form factor and the elastic form factor, respectively, where we study quite different parametrizations of the pion DA. Applying the conformal operator product expansion, which is valid in the CS scheme, we are able to predict the photon-to-pion transition form factor to next-to-next-to-leading order (NNLO) restricted to the conformal limit of the theory. At the moment evolution effects can not be taken into account at this order. Conclusions are given in Section VI. The decomposition and the conformal partial wave

expansion of functions appearing in the hard-scattering part of the electromagnetic form factor are listed in two Appendices.

II. SCHEME DEPENDENCE OF PQCD PREDICTIONS

In Ref. [14] we introduced the conformally covariant subtraction (CS) scheme, which ensures that the underlying conformal structure appearing in exclusive processes is manifest in the conformal limit. Calculations of hard-scattering amplitudes and evolution kernels are carried out usually in the $\overline{\mathrm{MS}}$ scheme. Both schemes are related to each other by a (finite) refactorization. For instance, the photon-to-pion transition form factor $F_{\gamma\pi}$ at large momentum transfer factorizes in both schemes according to the standard factorization scheme (SFA) [19,20]:

$$\begin{aligned} F_{\gamma\pi}(\omega, Q) &= T^{\mathrm{MS}}(\omega, x, Q, \mu) \otimes \phi^{\mathrm{MS}}(x, \mu) \\ &= T^{\mathrm{CS}}(\omega, x, Q, \mu) \otimes \phi^{\mathrm{CS}}(x, \mu), \qquad \otimes \equiv \int_0^1 dx, \end{aligned} \tag{1}$$

where the hard-scattering part $T(\omega, x, Q, \mu)$, depending on the kinematical variables ω and Q, the momentum fraction x as well as the factorization scale μ, can be calculated in perturbation theory. The DA $\phi(x, \mu)$ satisfy the Brodsky-Lepage (BL) evolution equation [21,19,13]:

$$\mu^2 \frac{d}{d\mu^2} \phi(x, \mu) = V(x, y; \alpha_s(\mu)) \otimes \phi(y, \mu). \tag{2}$$

Of course, the physical quantities are independent of the chosen scheme, which means that

$$\begin{aligned} T^{\mathrm{CS}}(\omega, x, Q, \mu) &= T^{\mathrm{MS}}(\omega, y, Q, \mu) \otimes B(y, x, \mu), \\ \phi^{\mathrm{CS}}(x, \mu) &= B^{-1}(x, y, \mu) \otimes \phi^{\mathrm{MS}}(y, \mu), \end{aligned} \tag{3}$$

where the B kernels satisfy $B(x, z, \mu) \otimes B^{-1}(z, y, \mu) = \delta(x - y)$. The latter transformation implies that the evolution kernel of the DA transforms inhomogeneously:

$$V^{\mathrm{CS}}(x, y) = B^{-1}(x, z) \otimes V^{\mathrm{MS}}(z, z') \otimes B(z', y) - \left[\mu^2 \frac{d}{d\mu^2} B^{-1}(x, z)\right] \otimes B(z, y). \tag{4}$$

Perturbative QCD predictions for physical quantities are given as truncated series in α_s. This necessary truncation induces their scheme dependence. The unknown higher order corrections may be minimized by choosing an appropriate scheme and scale. The problem of finding such an optimal renormalization scheme can be attacked with the help of the extended renormalization group equations introduced by Stückelberg and Peterman [22], which is equivalent to previous work given in Refs. [23–26]. To our best knowledge, no comparable methods were developed to find also the optimal factorization scheme. For a given process there may exist physical arguments to favour a special scheme. A further requirement should be that the factorization scheme respects the underlying symmetries of the theory, in order that no anomalous terms appear either in the hard-scattering part or in the BL evolution equation.

The more restricted problem to find the optimal scale in a given scheme has been widely discussed in the literature and three quite distinct methods have been proposed: the principle of fastest apparent convergence (FAC) [23,24], the principle of minimal sensitivity (PMS) [27–30], and the Brodsky-Lepage-Mackenzie (BLM) [31] scale setting. The application of these methods can yield quite different predictions (see for instance the analyses in Ref. [32]).

Although conformal symmetry holds only true in the hypothetical conformal limit, it should be manifested in the maximally possible manner in the full theory. For exclusive processes in which only mesons participate such a factorization scheme is, up to the scale setting problem, uniquely defined in the conformal limit[1]. However, as discussed above, also in the CS scheme anomalous terms proportional to the β-function are left and cannot, as we will see below, be uniquely fixed. According to the BLM scale setting prescription, these anomalous terms can be absorbed in the scale setting of the coupling, and the perturbative series will be formally the same as in the conformal theory. Motivated by our discussion it seems worthwhile to reexamine the known NLO corrections to exclusive processes in the CS scheme and to employ the BLM prescription for the absorption of the remaining conformal anomalies.

III. EVOLUTION OF THE FLAVOUR NON-SINGLET DISTRIBUTION AMPLITUDE

A. General formalism

First we discuss the application of the BLM scale fixing prescription in the evolution of the DA. For the convenience of the reader we outline the whole formalism for the solution of the evolution equation (2) in terms of the conformal partial wave expansion. Using the $\overline{\mathrm{MS}}$ scheme, the evolution kernel

$$V(x, y; \alpha_s) = \frac{\alpha_s}{2\pi} V^{(0)}(x, y) + \left(\frac{\alpha_s}{2\pi}\right)^2 V^{(1)}(x, y) + \cdots \tag{5}$$

was computed perturbatively in one- and two-loop approximation [34–37]. The one-loop kernel is diagonal with respect to Gegenbauer polynomials $C_k^{\frac{3}{2}}(2x-1)$ of order k and with index $3/2$. In the two-loop approximation this property is spoiled and because of the complicated structure, the moments

$$\gamma_{kn}(\alpha_s) = -2 \int_0^1 dx \int_0^1 dy\, C_k^{\frac{3}{2}}(2x-1) V(x, y; \alpha_s) \frac{(1-y)y}{N_n} C_n^{\frac{3}{2}}(2y-1), \tag{6}$$

where the normalization factor is $N_n = (n+1)(n+2)/(4(2n+3))$, cannot be directly calculated. Fortunately, one can use conformal constraints to compute the off-diagonal moments

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[1] In the case where baryons are also involved, it is known that the mixing problem in the evolution equation cannot be completely solved by conformal constraints [33], and thus also in this conformal subtraction scheme some freedom remains.

in a very economical way [38,16,17]. Indeed, in the $\overline{\rm MS}$ scheme the off-diagonal moments of the matrix $\hat\gamma$ are induced by a special conformal anomaly matrix $\hat\gamma^c$ and the running coupling [38,16,17]:

$$\left[\hat a(l) + \hat\gamma^c(l) + 2\frac{\beta}{g}\hat b(l), \hat\gamma\right] = 0, \tag{7}$$

where the matrices $\hat a$ and $\hat b$ have the following elements:

$$a_{kn}(l) = 2(k-l)(k+l+3)\delta_{kn}, \tag{8}$$

$$b_{kn}(l) = \begin{cases} 2(l+n+3)\delta_{kn} - 2(2n+3) & \text{if } k-n \geq 0 \text{ and even} \\ 0 & \text{otherwise.} \end{cases} \tag{9}$$

While the off-diagonal terms, induced by the renormalization of the coupling, are predicted by conformal constraints (7), the special conformal anomaly matrix $\hat\gamma^c$ contains new information. As explained in Refs. [39,38,16,17], this anomaly matrix can be calculated directly with the help of modified Feynman rules and it reads to leading order (LO):

$$\hat\gamma^{c(0)}(l) = -\hat b(l)\hat\gamma^{(0)} + \hat w,$$

where

$$w_{kn} = C_F \begin{cases} -4(2n+3)(k-n)(k+n+3)\times & \text{if } k-n>0 \\ \left[\frac{A_{kn}-\psi(k+2)+\psi(1)}{(n+1)(n+2)} + \frac{2A_{kn}}{(k-n)(k+n+3)}\right] & \text{and even} \\ 0 & \text{otherwise} \end{cases}, \tag{10}$$

$$A_{kn} = \psi\left(\frac{k+n+4}{2}\right) - \psi\left(\frac{k-n}{2}\right) + 2\psi\left(k-n\right) - \psi\left(k+2\right) - \psi(1),$$

with $\psi\left(z\right) = \frac{d}{dz}\ln\Gamma(z)$ and $C_F = 4/3$.

Employing the conformal partial wave expansion, which is given in terms of the eigenfunctions of the LO kernel $V^{(0)}(x,y)$,

$$\phi(x,Q) = {\sum_{k=0}^{\infty}}' \frac{(1-x)x}{N_k} C_k^{\frac{3}{2}}(2x-1)\langle 0|O_{kk}(\mu)|M(P)\rangle^{red}_{|\mu=Q}, \tag{11}$$

the BL evolution equation (2) can be perturbatively solved to any order [38]. Note that the expectation values of the operators for odd k vanish, which is indicated by the $\sum'$ symbol. The evolution of the composite operators appearing in Eq. (11) is governed by the renormalization group equation (RGE), which possesses the following triangular form due to Poincaré invariance:

$$\mu\frac{d}{d\mu}O_{kl} = -\gamma_k(\alpha_s(\mu))O_{kl} - {\sum_{n=0}^{k-2}}' \gamma_{kn}^{\rm ND}(\alpha_s(\mu))O_{nl}. \tag{12}$$

The off-diagonal matrix elements $\gamma_{kn}^{\rm ND}$ (with $k>n$) appear beyond the LO and are scheme dependent. In the $\overline{\rm MS}$ scheme they can be simple obtained from Eqs. (7)–(10).

In Ref. [14] it has been shown that in the conformal limit of the theory the CS scheme ensures the conformal covariance of the renormalized operators and, therefore, their anomalous-dimension matrix is diagonal. The transformation to the CS scheme is determined by a matrix $\hat B$, which depends only on $\hat\gamma^c$:

$$\hat{B} = \frac{\hat{1}}{\hat{1} + \mathcal{J}\hat{\gamma}^c} = \hat{1} - \mathcal{J}\hat{\gamma}^c + \mathcal{J}(\hat{\gamma}^c \mathcal{J}\hat{\gamma}^c) - \cdots, \tag{13}$$

where the operator $\mathcal{J}$ is defined by

$$\mathcal{J}\hat{A} := \begin{cases} \frac{A_{kn}}{2(k-n)(k+n+3)} & \text{if } k-n>0 \\ 0 & \text{otherwise.} \end{cases} \tag{14}$$

This transformation cancels the off-diagonal part of $\hat{\gamma}^{\mathrm{MS}}$, however, it induces an off-diagonal term proportional to the β-function:

$$\hat{\gamma}^{\mathrm{CS}} = \hat{B}^{-1}\hat{\gamma}^{\mathrm{MS}}\hat{B} - \beta\left[\frac{\partial}{\partial g}\hat{B}^{-1}\right]\hat{B}. \tag{15}$$

Hence, in this scheme the off-diagonal part of the anomalous-dimension matrix is proportional to the β-function:

$$\gamma^{\mathrm{ND}}_{kn} = \frac{\beta}{g}\Delta_{kn}, \quad \text{where } \Delta_{kn} = \frac{\alpha_s}{2\pi}\Delta^{(0)}_{kn} + O(\alpha_s^2). \tag{16}$$

The BLM scale setting prescription [31] can now be applied to absorb the off-diagonal term (16) into the scale dependence of the coupling:

$$\alpha_s(\mu^*) = \alpha_s(\mu)\left[1 - \beta_0\frac{\alpha_s(\mu)}{2\pi}\ln\left(\frac{\mu^*}{\mu}\right) + \cdots\right]. \tag{17}$$

Thus, the off-diagonal term in NLO may be expressed as

$$\begin{aligned}\gamma^{\mathrm{ND}}_{kn} &= -\left(\frac{\alpha_s(\mu)}{2\pi}\right)^2\beta_0\Delta^{(0)}_{kn} + \cdots = \left[\frac{\alpha_s(\mu^*_{kn})}{2\pi} - \frac{\alpha_s(\mu)}{2\pi}\right]\gamma^{(0)}_k + \cdots \\ &= \gamma_k(\alpha_s(\mu^*_{kn})) - \gamma_k(\alpha_s(\mu)) + \cdots,\end{aligned} \tag{18}$$

where $\mu^*_{kn} = \mu\exp\left\{\Delta^{(0)}_{kn}/\gamma^{(0)}_k\right\}$ is the new scale. To absorb the NNLO corrections in the analogous way, it is necessary to introduce a different second scale μ^{**}_{kn} and so on. After this procedure the RGE (19) in the CS scheme takes the form:

$$\mu\frac{d}{d\mu}O^{\mathrm{co}}_{kl} = -\gamma_k(\alpha_s(\mu))O^{\mathrm{co}}_{kl} - \sum_{n=0}^{k-2}{}'\left[\gamma_k(\alpha_s(\mu^*_{kn}), \alpha_s(\mu^{**}_{kn}), \ldots) - \gamma_k(\alpha_s(\mu))\right]O^{\mathrm{co}}_{nl}. \tag{19}$$

All scales $\mu^*_{kn}, \mu^{**}_{kn}, \ldots$ are uniquely determined by the conformal symmetry breaking term Δ_{kn}. This anomaly arises from two sources: (i) from the renormalization of the coupling constant that enters in the anomalous-dimension matrix and (ii) from the renormalization prescription in the CS scheme.

Here one comment is in order. Since the effects of arbitrary renormalization transformations proportional to the β function will disappear in the conformal limit, the CS scheme cannot be fixed in the full theory. Below we will deal with two such CS schemes referred as $\mathrm{CS_I}$ and $\mathrm{CS_{II}}$. In the first one only the off-diagonal part that is related to the special-conformal anomaly matrix will be removed by the transformation (15), while in the $\mathrm{CS_{II}}$

scheme also the off-diagonal terms proportional to β_0 appearing in the anomalous dimensions are explicitly removed (obviously, the inhomogeneous part in Eq. (15) cannot be avoided, and finally terms proportional to β_0 appear). Although the $\mathrm{CS_{II}}$ scheme induces an additional symmetry breaking term proportional to the β-function in the hard-scattering part, which looks artificial, it will be interesting to compare both CS schemes with each other.

The evolution equation (2) can be perturbatively solved with the help of the conformal spin expansion. Solving the RGE (12), which is an inhomogeneous partial first order differential equation, the solution has been written in a compact form in Ref. [38] and it is valid for an arbitrary scheme:

$$\phi(x,Q^2) = {\sum_{n=0}^{\infty}}' \varphi_n(x,Q,Q_0) \exp\left\{-\int_{Q_0}^{Q} \frac{d\mu}{\mu}\gamma_n(\mu)\right\} \langle 0|O_{nn}(Q_0)|M(P)\rangle^{red}. \tag{20}$$

The partial waves $\varphi_n(x,Q,Q_0)$ now contain perturbative corrections, which are induced by the off-diagonal anomalous dimension matrix. They are also known as expansion with respect to the Gegenbauer polynomials

$$\varphi_n(x,Q,Q_0) = {\sum_{k=n}^{\infty}}' \frac{(1-x)x}{N_k} C_k^{\frac{3}{2}}(2x-1) B^{\mathrm{dia}}_{kn}(Q,Q_0). \tag{21}$$

The matrix $B^{\mathrm{dia}}_{kn}(Q,Q_0)$ diagonalizes the RGE (12) of the conformal operators and is given by

$$\hat{B}^{\mathrm{dia}} = \frac{\hat{1}}{\hat{1}-\mathcal{L}\hat{\gamma}^{\mathrm{ND}}} = \hat{1} + \mathcal{L}\hat{\gamma}^{\mathrm{ND}} + \mathcal{L}\left(\hat{\gamma}^{\mathrm{ND}}\mathcal{L}\hat{\gamma}^{\mathrm{ND}}\right) + \cdots, \tag{22}$$

where the operator $\mathcal{L}$ is an integral operator acting on a triangular and off-diagonal matrix:

$$\mathcal{L}\gamma^{\mathrm{ND}}_{kn} = -\int_{Q_0}^{Q} \frac{d\mu}{\mu}\gamma^{\mathrm{ND}}_{kn}(\mu) \exp\left\{-\int_{\mu}^{Q}\frac{d\mu'}{\mu'}\left[\gamma_k(\mu')-\gamma_n(\mu')\right]\right\}. \tag{23}$$

Here we do not include radiative corrections at the reference point Q_0, i.e. $B^{\mathrm{dia}}_{kn}(Q_0,Q_0) = \delta_{kn}$. Notice that in general the matrix $\hat{B}^{\mathrm{dia}}$ is different from the previously introduced matrix $\hat{B}$.

B. NLO analysis

In the $\overline{\mathrm{MS}}$ scheme the evolution of the DA in NLO was analysed in detail in Ref. [40]. The main feature showing up in this order is the excitation of higher harmonics due to the mixing of the operators, which yields logarithmic corrections in the end-point region. This logarithmical enhancement is hidden in the expansion of the partial waves, which is, corresponding to Eqs. (22)-(21), given as

$$\varphi_n(x,Q,Q_0) = \frac{(1-x)x}{N_n} C_n^{\frac{3}{2}}(2x-1) + \frac{\alpha_s(Q)}{2\pi}\varphi_n^{(1)}(x,Q,Q_0) + \cdots, \tag{24}$$

$$\varphi_n^{(1)}(x,Q,Q_0) = {\sum_{k=n+2}^{\infty}}' \frac{(1-x)x}{N_k} C_k^{\frac{3}{2}}(2x-1) \frac{1-\left(\frac{\alpha_s(Q_0)}{\alpha_s(Q)}\right)^{\frac{\beta_0+\gamma_n^{(0)}-\gamma_k^{(0)}}{\beta_0}}}{\beta_0+\gamma_n^{(0)}-\gamma_k^{(0)}}\gamma_{kn}^{(1)\mathrm{ND}}.$$

The off-diagonal matrix $\hat{\gamma}^{(1)\mathrm{ND}}$ is determined by the conformal anomalies and in the $\overline{\mathrm{MS}}$ scheme it can be easily obtained from Eq. (7):

$$\gamma_{kn}^{(1)\mathrm{ND}} = (\gamma_k^{(0)} - \gamma_n^{(0)}) \frac{\gamma_{kn}^{c(0)} - \beta_0 b_{kn}}{2(k-n)(k+n+3)}. \tag{25}$$

Here the matrices $\hat{b}$ and $\hat{\gamma}^c$ are defined in Eqs. (9) and (10), respectively.

Above we introduced two conformal schemes $\mathrm{CS_I}$ and $\mathrm{CS_{II}}$ which are obtained from the $\overline{\mathrm{MS}}$ one by the transformations

$$\begin{aligned} B_{kn} &= \delta_{kn} - \frac{\alpha_s}{2\pi} \frac{\gamma_{kn}^{c(0)}}{2(k-n)(k+n+3)} \quad \text{for } \mathrm{CS_I}, \\ B_{kn} &= \delta_{kn} - \frac{\alpha_s}{2\pi} \frac{\gamma_{kn}^{c(0)} - \beta_0 b_{kn}}{2(k-n)(k+n+3)} \quad \text{for } \mathrm{CS_{II}}, \end{aligned} \tag{26}$$

respectively. According to Eq. (15) these transformations imply that the off-diagonal part is $\gamma_{kn}^{(1)\mathrm{ND}} = -\beta_0 \Delta_{kn}^{(0)}$ [see Eq. (16)] with

$$\begin{aligned} \Delta_{kn}^{(0)} &= \frac{(\gamma_k^{(0)} - \gamma_n^{(0)}) b_{kn} - \gamma_{kn}^{c(0)}}{2(k-n)(k+n+3)} \quad \text{for } \mathrm{CS_I}, \\ \Delta_{kn}^{(0)} &= \frac{\beta_0 b_{kn} - \gamma_{kn}^{c(0)}}{2(k-n)(k+n+3)} \quad \text{for } \mathrm{CS_{II}}. \end{aligned} \tag{27}$$

$\Delta_{kn}^{(0)}$ can be absorbed via the BLM scale setting prescription; so that then the partial waves read

$$\begin{aligned} \varphi_n^{\mathrm{CS}}(x, Q, Q_0) = {} & \frac{(1-x)x}{N_n} C_n^{\frac{3}{2}}(2x-1) + \sum_{k=n+2}^{\infty}{}' \frac{(1-x)x}{N_k} C_k^{\frac{3}{2}}(2x-1) \gamma_k^{(0)} \left(1 - \frac{\alpha_s(Q)}{\alpha_s(Q_{kn}^*)}\right) \\ & \times \frac{1 - \left(\frac{\alpha_s(Q_0)}{\alpha_s(Q)}\right)^{\frac{\beta_0 + \gamma_n^{(0)} - \gamma_k^{(0)}}{\beta_0}}}{\beta_0 + \gamma_n^{(0)} - \gamma_k^{(0)}} + \cdots . \end{aligned} \tag{28}$$

Finally, we study in the $\overline{\mathrm{MS}}$ and in both CS schemes the scale dependence of the quantity

$$I(Q, Q_0) = \int_0^1 dx \, \frac{\varphi(x, Q, Q_0)}{x}, \tag{29}$$

which enters in different exclusive mesonic processes. For convenience we changed the normalization of the DA, i.e. $\phi(x) = f_\pi \varphi(x)/2\sqrt{6}$, where the pion decay constant is $f_\pi \approx 0.131$ GeV. Notice that this integral was originally defined to LO; here, however, it also contains the higher order effects caused by the evolution. It is very sensitive to the end-point behaviour of the DA and, therefore, it may serve as a measure for the logarithmic corrections due to the evolution, which occur in this region [40].

Inserting the conformal spin expansion (20) into the integral (29) provides the representation

$$I(Q,Q_0)=\sum_{n=0}^{\infty}{}' I_n(Q,Q_0)\exp\left\{-\int_{Q_0}^{Q}\frac{d\mu}{\mu}\gamma_n(\mu)\right\}\langle 0|O_{nn}(Q_0)|\pi(P)\rangle^{red}, \tag{30}$$

$$I_n(Q,Q_0)=\frac{2(3+2n)}{(n+1)(n+2)}+\frac{\alpha_s(Q)}{2\pi}\sum_{k=n+2}^{\infty}{}'\frac{2(3+2k)}{(k+1)(k+2)}\gamma_{kn}^{(1)\mathrm{ND}} \times\frac{1-\left(\frac{\alpha_s(Q_0)}{\alpha_s(Q)}\right)^{\frac{\beta_0+\gamma_n^{(0)}-\gamma_k^{(0)}}{\beta_0}}}{\beta_0+\gamma_n^{(0)}-\gamma_k^{(0)}}+\cdots, \tag{31}$$

which will be evaluated numerically by an appropriate truncation of the series. Here the reduced matrix elements are the conformal moments of the DA now normalized as:

$$\langle 0|O_{nn}(\mu)|\pi(P)\rangle^{red}=\int_0^1 dx\, C_n^{\frac{3}{2}}(2x-1)\varphi(x,\mu). \tag{32}$$

It is worthy to mention that in the CS schemes, the direct integration of the RG equation (19) implies the following form:

$$I_n(Q,Q_0)=\frac{2(3+2n)}{(n+1)(n+2)}\left(1+1-\frac{\alpha_s(Q)}{\alpha_s(\overline{Q}_n)}+\cdots\right), \tag{33}$$

where $\overline{Q}_n$ depends on Q and Q_0:

$$1-\frac{\alpha_s(Q)}{\alpha_s(\overline{Q}_n)}=\frac{(n+1)(n+2)}{2(3+2n)}\sum_{k=n+2}^{\infty}{}'\frac{2(3+2k)}{(k+1)(k+2)}\gamma_k^{(0)}\left(1-\frac{\alpha_s(Q)}{\alpha_s(Q^*_{kn})}\right) \times\frac{1-\left(\frac{\alpha_s(Q_0)}{\alpha_s(Q)}\right)^{\frac{\beta_0+\gamma_n^{(0)}-\gamma_k^{(0)}}{\beta_0}}}{\beta_0+\gamma_n^{(0)}-\gamma_k^{(0)}}. \tag{34}$$

Notice that in the conformal scheme $\mathrm{CS_{II}}$ a contribution of the form $1-\alpha_s(Q)/\alpha_s(\overline{Q})$ arises in the hard-scattering amplitudes, too. Obviously, their expansion gives an α_s-suppressed term proportional to β_0.

Taking into account a sufficient large number of terms in the series (30) – (33) (details for the estimate of the accuracy of such an approximation can be found in Ref. [40]), the relative correction to NLO, i.e.

$$I^{\mathrm{rel}}(Q,Q_0)=\frac{I^{\mathrm{NLO}}(Q,Q_0)-I^{\mathrm{LO}}(Q,Q_0)}{I^{\mathrm{LO}}(Q,Q_0)}, \tag{35}$$

is numerically evaluated, where we consequently took into account the following perturbative expansion:

$$\exp\left\{-\int_{Q_0}^{Q}\frac{d\mu}{\mu}\gamma_n(\mu)\right\}=\left(\frac{\alpha_s(Q)}{\alpha_s(Q_0)}\right)^{\frac{\gamma_n^{(0)}}{\beta_0}}\left[1+\left(\frac{\alpha_s(Q)}{2\pi}-\frac{\alpha_s(Q_0)}{2\pi}\right)\left(\frac{\gamma_n^{(1)}}{\beta_0}-\frac{\beta_1}{2\beta_0}\frac{\gamma_n^{(0)}}{\beta_0}\right)+\cdots\right]. \tag{36}$$

Here $\beta_0=11-2/3n_f$, $\beta_1=102-38/3n_f$, $\gamma_n^{(0)}=-C_F\{3+1/[(n+1)(n+2)]-4\psi(n+2)+4\psi(1)\}$, and the expression for $\gamma_n^{(1)}$ can be found for instance in Ref. [41]. Two values of the

QCD scale parameter, a rather small one $\Lambda^{\rm LO} = 100$ MeV as well as a rather large one $\Lambda^{\rm LO} = 500$ MeV are chosen. Since in general non-perturbative (model) calculations are performed at a low scale, we choose the reference momentum $Q_0 = \sqrt{0.5}$ GeV. In the following it is assumed that the evolution equation can be used at this low scale. The evolution runs up to the scale $Q = \sqrt{20}$ GeV. The number of active quarks is $n_f = 3$ and we take the following set of DAs: asymptotic, two-hump and $\varphi^a(x) = \Gamma(2a+2)/\Gamma(a+1)^2[x(1-x)]^a$ with $a = \{10, 1/2, 1/4\}$, i.e. very narrow and broad ones are also included. The resulting relative NLO corrections in percentage are listed for the three considered schemes in Table I.

		$\Lambda = 100$ MeV			$\Lambda = 500$ MeV		
DA	$I(Q_0, Q_0)$	$\overline{\rm MS}$	$\rm CS_I$	$\rm CS_{II}$	$\overline{\rm MS}$	$\rm CS_I$	$\rm CS_{II}$
φ^{10}	2.1	0.7%	1.0%	1.0%	1.6%	4.2%	2.2%
$\varphi^{\rm as}$	3.0	-0.1%	0.4%	-0.4%	-1.3%	2.3%	-3.0%
$\varphi^{1/2}$	4.0	-0.8%	-0.3%	-2.4%	-3.1%	0.7%	-7.8%
$\varphi^{\rm CZ}$	5.0	-1.2%	-0.4%	-2.5%	-5.6%	-0.6%	-11.7%
$\varphi^{1/4}$	6.0	-1.2%	-2.3%	-6.6%	-6.7%	-1.6%	-25.%

TABLE I. The value of the I integral at the input scale $Q_0 = \sqrt{0.5}$ GeV and its relative NLO corrections at $Q = \sqrt{20}$ GeV are listed for different DAs in the $\overline{\rm MS}$, $\rm CS_I$, and $\rm CS_{II}$ scheme for two choices of the QCD scale parameter $\Lambda^{\rm LO} = 100,\ 500$ MeV.

For both the narrow and the asymptotic DA the NLO corrections are rather small in all three schemes. The corrections are negative for broader DAs and their absolute value is increasing with growing value of $I^{\rm LO}$, however, they remain rather small in the $\rm CS_I$ scheme. In the $\rm CS_{II}$ scheme the corrections are larger than in the $\overline{\rm MS}$ scheme, so one may conclude that this conformal scheme is disfavoured. Let us mention that in the $\overline{\rm MS}$ scheme the contributions from the off-diagonal part are of the same sign as the α_s^2 corrections to the diagonal part of the anomalous-dimension matrix and that both of them enter in the relative NLO contribution with a similar size. In the $\overline{\rm MS}$ and the $\rm CS_I$ scheme, but not in the $\rm CS_{II}$ one, a partial cancellation in the off-diagonal terms takes place between the special conformal anomaly and the β_0 term [40], and that the net contribution is small. Note that for a reference momentum of 1 GeV or even larger the size of the NLO corrections decreases. We may conclude that in general the NLO corrections due to the evolution of the DA are small, except for very broad DA evolved in the $\rm CS_{II}$ scheme.

IV. PHOTON-TO-PION TRANSITION FORM FACTOR

From the theoretical point of view the simplest mesonic process is the production of a pseudoscalar meson in two-photon collisions

$$\gamma^*(q_1)\,\gamma^*(q_2) \to M(P), \tag{37}$$

since it is purely electromagnetic to LO. As independent kinematical variables we choose the negative of the momentum transfer between the photons squared $Q^2 = -q^2$, with $q = (q_1 - q_2)/2$ and the asymmetry parameter $\omega = Pq/Q^2$. In the case that one photon is on mass-shell we have $|\omega| = 1$, while for equal photon virtualities $|\omega| = 0$. The dynamical information is contained in the amplitude $\Gamma_{\alpha\beta} = -ie^2 \epsilon_{\alpha\beta\mu\nu} q_1^\mu q_2^\nu F_{\gamma M}(\omega, Q^2)$, where the photon-to-meson transition form factor $F_{\gamma M}(\omega, Q^2)$ is defined in terms of the time ordered product of two electromagnetic currents sandwiched between the one-meson state and the vacuum:

$$- ie^2 \epsilon_{\alpha\beta\mu\nu} q_1^\mu q_2^\nu F_{\gamma M}(\omega, Q^2) = i \int d^4x \, e^{ixq} \left\langle M \middle| T J^\mu \left(\frac{x}{2}\right) J^\nu \left(-\frac{x}{2}\right) \middle| 0 \right\rangle. \tag{38}$$

At large momentum transfer this transition form factor has been measured for π^0, η, and η' mesons in single antitagged experiments by the CELLO collaboration [42] and, more recently, at CLEO [43,44], where the untagged photon is almost real. The photon-to-pion transition form factor has been determined for $0.5 \le Q^2 \le 2.7\ \mathrm{GeV}^2$ [42] and $1.5 \le Q^2 \le 9\ \mathrm{GeV}^2$, respectively [43,44]. For the CLEO data the virtuality of the second photon was estimated to be less than $0.001\ \mathrm{GeV}^2$. The η and η' transition form factors are known up to $20\ \mathrm{GeV}^2$ and $30\ \mathrm{GeV}^2$, respectively [43,44] (the systematic errors become very large with increasing photon virtuality). Data at lower momentum transfer are given in Refs. [45,46].

Taking the SFA to LO [19,20], the normalization of the photon-to-pion transition form factor is consistent with pion DA's that are *not* concentrated in the end-point region. Perturbative and non-perturbative corrections to this prediction have been studied in a number of papers. Let us only mention that non-perturbative effects have been included in a model dependent way by the transverse momentum dependence [47–50] or by the sum rule approach [51–53]. All these analyses show that the data can be reproduced by a DA that is close to the asymptotic one[2]: $\varphi^{\mathrm{as}} = 6x(1-x)$. It is maybe interesting to note that the authors in Ref. [50] observed that, taking into account the transverse momentum dependence in the light-cone formalism, quite different model wave functions are consistent with the data. In the following we rely on the SFA, which allows us to calculate the perturbative corrections in a systematic way [19]. From this factorization procedure it is expected that large soft corrections from the transverse momentum dependence in the pion wave function would induce large perturbative corrections in the SFA.

Alternative to the SFA the transition form factor at large momentum transfer can be calculated with the help of the operator product expansion (OPE) at light-like distances. In a conformal invariant theory, the form of the Wilson coefficients are fixed up to the normalization [54,55]. In Ref. [14] it has been shown that this conformally covariant OPE, holds true for $\beta = 0$ in the CS scheme. Taking into account the general structure of this conformal prediction, the decomposition of the transition form factor in conformal partial waves is given in the conformal limit as:

---

[2]In the prediction of the transition form factor only the integral $\int_0^1 dx\, \varphi(x)/x$ enters and its value extracted from the data is of about 3 or even smaller. Of course, the asymptotic DA provides 3, however, we should mention that one cannot conclude in the mathematical sense that its shape is convex; even if one takes into account normalization and positivity.

$$F_{\gamma\pi}(\omega,Q) = \frac{2\sqrt{2}f_\pi}{3Q^2}\sum_{k=0}^{\infty}{}' B(k+1,k+2)c_k(\alpha_s(\mu))\left(\frac{\mu^2}{(1+\omega)Q^2}\right)^{\frac{\gamma_k}{2}}\frac{2(2\omega)^k}{(1+\omega)^{k+1}} \qquad (39)$$

$$\times {}_2F_1\left(\begin{matrix} k+1+\frac{1}{2}\gamma_k, k+2+\frac{1}{2}\gamma_k \\ 2(k+2+\frac{1}{2}\gamma_k)\end{matrix}\middle| \frac{2\omega}{1+\omega}\right)\langle\pi(P)|O^{\rm co}_{kk}(\mu)|0\rangle^{red}.$$

The Wilson coefficients c_k and the anomalous dimensions γ_k are known up to order α_s^2 from the perturbative corrections to the longitudinal structure function g_1 [56,57]. Here the reduced matrix elements are normalized according to Eq. (32) and c_k are equal to one in LO.

As already mentioned, the pion transition form factor has been measured by single antitagged experiments in the region of $0.5 - 9$ GeV^2 at CELLO and CLEO, where the antitagged photon is almost on-shell, so that we can set $\omega = 1$. Hence, employing the relation ${}_2F_1(a,b,c|1) = \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}$ leads to a considerable simplification of the prediction:

$$F_{\gamma\pi}(\omega=1,Q^2) = \frac{2\sqrt{2}f_\pi}{3Q^2}\sum_{k=0}^{\infty}{}'\frac{\Gamma(k+1)\Gamma(k+2)\Gamma\left(2k+4+\gamma_k\right)}{\Gamma\left(k+2+\frac{1}{2}\gamma_k\right)\Gamma\left(k+3+\frac{1}{2}\gamma_k\right)\Gamma\left(2k+3\right)} \qquad (40)$$

$$\times c_k(\alpha_s(\mu))\left(\frac{\mu^2}{-q_1^2}\right)^{\frac{\gamma_k}{2}}\langle\pi(P)|O^{\rm co}_{kk}(\mu^2)|0\rangle^{red}.$$

A. NLO corrections

It is again worthwhile to mention that in the favoured $\mathrm{CS_I}$ scheme the prediction (40) is exact to NLO, while in the $\overline{\mathrm{MS}}$ scheme and the $\mathrm{CS_{II}}$ scheme it is spoiled by the special conformal anomaly and a term proportional to β_0, respectively. To evaluate the formula (40) to NLO in the $\mathrm{CS_I}$ scheme, the right-hand side (RHS) of Eq. (40) is consequently expanded up to the order α_s. The NLO results in the two other schemes are easily obtained by employing the transformations (26). Table II contains the absolute and relative α_s corrections to the hard-scattering amplitude, where the virtuality of the space-like photon is $-q_1^2 = 2$ GeV^2 and the QCD scale parameter is set to $\Lambda^{\rm LO} = 0.22$ GeV. For simplicity, we use the "natural" scale setting prescription $\mu^2 = -q_1^2$ in Eq. (40) [see the μ dependence of the coefficient function] and do not discuss the scale setting dependence. The set of DAs is the same as before, however, now at the input scale of $\sqrt{2}$ GeV instead of 0.5 GeV, with the exception that the two-hump function is evolved from its normalization point at 0.5 GeV to $\sqrt{2}$ GeV [58]. For the narrow DA the NLO correction is in all schemes surprisingly large, namely, between -25 to -31%. For the asymptotic DA the NLO correction is about -20% in both the $\overline{\mathrm{MS}}$ and the $\mathrm{CS_{II}}$ scheme, while it is only -12% in the $\mathrm{CS_I}$ scheme. A similar reduction of about 10% in the favoured $\mathrm{CS_I}$ scheme is observed in the case of the two-hump DA resulting in negligible small perturbative corrections in this case. For convex amplitudes that are strongly concentrated in the end-point region the corrections are positive and becoming very large. These results are due to the fact that *only* the *first* conformal partial wave has a negative contribution.

Here one comment is in order. For a *given* DA, the differences of the predictions in Table II do *not* reflect the scheme dependence of the NLO result. One has also to take into account

DA	LO	NLO					
		$\overline{\mathrm{MS}}$	$\mathrm{CS_I}$	$\mathrm{CS_{II}}$	$\overline{\mathrm{MS}}$	$\mathrm{CS_I}$	$\mathrm{CS_{II}}$
φ^{10}	0.13	0.09	0.10	0.09	-31%	-25%	-28%
φ^{as}	0.19	0.15	0.16	0.14	-20%	-12%	-21%
$\varphi^{1/2}$	0.25	0.26	0.27	0.24	5%	11%	-5%
φ^{CZ}	0.26	0.24	0.26	0.22	-10%	-1%	-14%
$\varphi^{1/4}$	0.37	0.78	0.76	0.63	110%	104%	70%

TABLE II. Absolute LO and NLO predictions for the photon-to-pion transition form factor and their relative deviation are listed for different DAs. Here the scale Q is set to $\sqrt{2}$ GeV and the evolution of the DA is taken into account only for the two-hump function.

the transformation (1) of the DA, which would reduce the observed differences drastically. For instance, if we take the asymptotic DA in the $\mathrm{CS_I}$ scheme, the input in the $\overline{\mathrm{MS}}$ scheme is [40]

$$\varphi^{\mathrm{MS}}(x) = B \otimes 6x(1-x) = 6x(1-x)\left\{1 + \frac{\alpha_s}{4\pi} C_F \left[\ln^2\left(\frac{1-x}{x}\right) + 2 - \frac{\pi^2}{3}\right]\right\}. \qquad (41)$$

The transformation from a given scheme to a second one leads to a logarithmic modification of the DA in order α_s. In fact the problem arises to which scheme the used non-perturbative input is related. Although in the sum rule approach radiative corrections are considered as unimportant, for this problem in question, it is interesting to study such perturbative corrections. It is expected that radiative corrections are minimized in the $\mathrm{CS_I}$ scheme.

Now we would like to discuss shortly the radiative corrections in their dependence on ω. Since $F_{\gamma\pi}(\omega, Q)$ is symmetric with respect to $\omega \to -\omega$, it is sufficient to consider $0 \le \omega \le 1$. From Eq. (39) it is obvious that for $\omega < 1$ the conformal partial waves for $2 \le k$ are suppressed by ω^k. For $\omega = 0$ only the first term contributes. The first moment of the DA is uniquely given in terms of the pion decay constant; moreover, the anomalous dimension $\gamma_0 = 0$, so the radiative correction, which is also independent of the DA, is determined only by the α_s corrections to e_0. In the conformal limit of the theory, we may conclude from Eq. (39) that the ω dependence of the perturbative correction to the conformal partial waves is rather smooth.

In Fig. 1(a) the SFA predictions to NLO are compared to the experimental data [42–44] for the narrow, asymptotic, and two-hump DA in both the $\overline{\mathrm{MS}}$ and $\mathrm{CS_I}$ scheme. The reference momentum for all DAs is now set to $Q_0 = 0.5$ GeV and $\Lambda^{\mathrm{LO}} = 220$ MeV. As pointed out above the corrections in the conformal subtraction scheme $\mathrm{CS_I}$ are about 10% smaller than in the $\overline{\mathrm{MS}}$ one. Caused by the NLO corrections the prediction of the asymptotic DA now agrees better with the data, while the prediction of the narrow DA starts to be below the data. In both schemes the two-hump DA remains incompatible with the data. However, it can be demonstrated that its prediction can be pushed down for a suitable choice of parameters. For instance, if we choose the $\mathrm{CS_{II}}$ scheme and $\Lambda^{\mathrm{LO}} = 330$ MeV the predictions of all three DAs start to be compatible with the data as illustrated in Fig. 1. This is qualitatively the same effect as observed in Ref. [50] in the context of the model dependent

study of the transverse momentum dependence. Since this scheme transformation does not look very natural, we cannot draw any conclusions, however, we may get some motivation to study perturbative corrections beyond the NLO. Also we would like to note again that as discussed above, the differences in the predictions arise mainly from the fact that we "forget" to transform the DA.

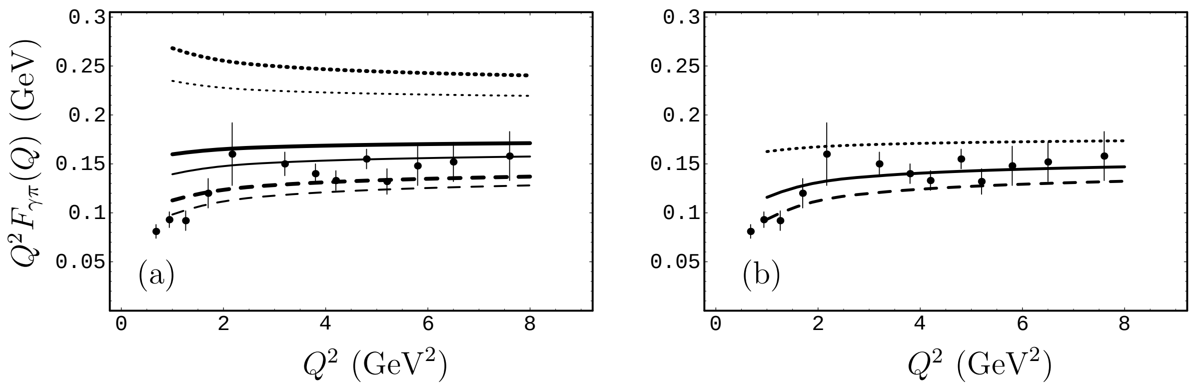

FIG. 1. The SFA predictions for the pion to photon form factor to NLO are shown for the narrow (dashed line), asymptotic (solid line), and two-hump (dotted line) DA, where the reference momentum is $Q_0 = 0.5$ GeV. In (a) the predictions in the $\overline{\mathrm{MS}}$ and $\mathrm{CS_I}$ schemes are represented as thin and thick lines, respectively, where $\Lambda^{\mathrm{LO}} = 220$ MeV. In (b) the $\mathrm{CS_{II}}$ scheme is chosen and $\Lambda^{\mathrm{LO}} = 330$ MeV.

B. Beyond NLO

As we can imagine from the results given in the previous subsection, for the phenomenology of the considered transition form factor it is an important task to study the corrections beyond the NLO. However, such calculations have not been carried out at present. Fortunately, we can rely on the conformal prediction (39), valid for $\beta = 0$, where the Wilson coefficients and the anomalous dimensions are known up to order α_s^2 from the results for the longitudinal structure function g_1 in DIS [56,57]. At NNLO it is known that the result (40) is also spoiled in the $\mathrm{CS_I}$ scheme by a term proportional to the β-function [59,18]. For these restricted predictions, symmetry breaking terms proportional to the β coefficient are neglected. Especially, for the asymptotic DA the NNNLO correction is available from the Bjorken sum rule.

The photon-to-pion transition form factor predicted by the asymptotic DA is given by the first term of the conformal expansion (39):

$$Q^2 F_{\gamma\pi}(\omega, Q^2) = \frac{\sqrt{2} f_\pi}{3} \frac{2}{1+\omega} \, {}_2F_1\left(\begin{matrix} 1,2 \\ 4 \end{matrix} \middle| \frac{2\omega}{1+\omega} \right) c_0(\alpha_s). \tag{42}$$

The coefficient $c_0(\alpha_s)$ is normalized to 1 at LO. For the case that one photon is almost real, i.e. $\omega = 1$, we get

$$Q^2F(1,Q^2) = \sqrt{2}f_\pi c_0(\alpha_s) = 0.185\, c_0(\alpha_s)\ \mathrm{GeV}. \qquad (43)$$

In the conformal limit the first moment and thus also the asymptotic DA do not evolve. The predictive power of the conformally covariant operator product expansion (OPE) tells us that the coefficient $c_0(\alpha_s)$ is the value of the Bjorken sum rule, which is calculated up to order α_s^3 [60,61]. For three active flavours the numerical result reads[3]

$$c_0(\alpha_s) = 1 - \frac{\alpha_s}{\pi} - 3.58333\left(\frac{\alpha_s}{\pi}\right)^2 - 20.21527\left(\frac{\alpha_s}{\pi}\right)^3 + O\left(\alpha_s^4\right). \qquad (44)$$

The higher-loop corrections at a scale of $Q^2 = 2\ \mathrm{GeV}^2$, where Λ^{LO} is assumed again to be 220 MeV, reduce the LO prediction to about 17% in NNLO and to about 20% in NNNLO (the NLO contribution is 12%).

From the size of the evolution effects arising in NLO we suspect that the NNLO corrections remain small and can be neglected in a first step. To obtain the NNLO corrections to the hard-scattering part for general DAs, Eq. (40) is expanded up to order α_s^2. In the case of the two-hump DA (again evolved to a scale of $Q = \sqrt{2}$ GeV) the NNLO correction remains negligibly small, namely about -2%. For the chosen narrow DA the correction decreases from about -24% in NLO to about -30% in NNLO.

We may conclude that in the conformal limit of the theory the perturbative series looks very reasonable for DAs that are not ruled out by the data. It would be very interesting to calculate the symmetry breaking effects in NNLO, which can be done by taking into account only the n_f dependent part of the gluon vacuum polarization, which arise from the quark loop. Indeed, this calculation has been carried out recently in the $\overline{\mathrm{MS}}$ scheme for the hard scattering part [59,18], however, the given representation does not allow to transform this result to the $\mathrm{CS_I}$ scheme in a closed form. Moreover, to be sure that the n_f dependent term belongs to a conformal anomaly proportional to the β_0 term, we have to combine the n_f part of the hard-scattering amplitude with that of the special conformal anomaly to NLO. If one had at hand the desired corrections to the conformal prediction (39), one would be able to fix the scale by the BLM scale setting prescription.

V. PION FORM FACTOR

The space-like elastic pion form factor has been extracted for $0.25 \le Q^2 \le 6\ \mathrm{GeV}^2$ from the measured cross section of the process $\gamma^* p \to \pi^+ n$. The intermediate off mass-shell pion was extrapolated to the pion pole [64,65]. Since this procedure suffers from large systematic errors [66], a direct measurement of the form factor would be very desirable. Indeed, the debate concerning the applicability of the SFA to the electromagnetic pion form factors is based on these data, which are doubted in Refs. [66,67]. In accordance with the counting rule, the data behave as $1/Q^2$ for $Q^2 \ge 1\ \mathrm{GeV}^2$. In the SFA the form factor is predicted to LO as:

$$F_\pi(Q) = \frac{2\pi f_\pi^2 C_F \alpha_s(Q)}{3Q^2} I_0(Q)^2, \quad I_0(Q) = \int_0^1 dx\, \frac{\varphi(x,Q)}{1-x}. \qquad (45)$$

[3] The α_s^4-correction has been estimated to be negative, too [62,63].

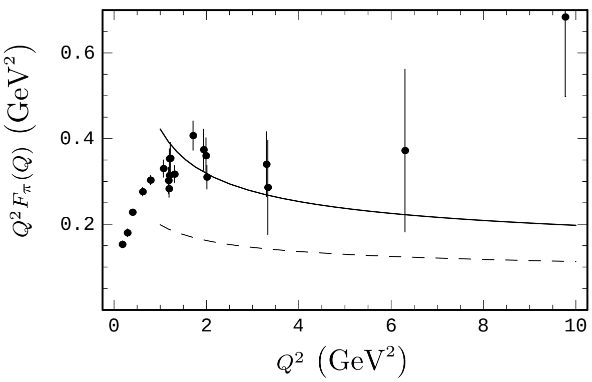

FIG. 2. Predictions for the electromagnetic pion form factor at large momentum transfer Q for the asymptotic DA (dashed line) and the two-hump DA (solid line) to LO, where $\Lambda_{\rm QCD} = 220$ MeV. Data are taken from Ref. [64] and references therein.

To fit the existing data shown in Fig. 2, the value of I_0 should be about 4-5, so that an end-point enhanced DA such as the two-hump function seems to be preferred.

The general factorization formula for the electromagnetic pion form factor at large momentum transfer reads

$$F_\pi(Q) = \phi(x,\mu_F,\mu_R) \otimes T(x,y,Q,\mu_F,\mu_R) \otimes \phi(y,\mu_F,\mu_R), \tag{46}$$

where the scales are distinguished as the factorization scale μ_F and the renormalization scale μ_R. Note that also the DA depends on the renormalization scale μ_R. However, for simplicity we will not discuss this dependence in detail and take into account evolution effects by the solution of Eq. (2), in which both scales are identified. The α_s correction to the hard-scattering part

$$T(x,y,Q^2) = \frac{16\pi C_F \alpha_s(\mu_R)}{xyQ^2}\left[1 + \frac{\alpha_s(\mu_R)}{2\pi} T^{(1)}(x,y,Q,\mu_F,\mu_R) + O\left(\alpha_s^2\right)\right] \tag{47}$$

was calculated by different authors in dimensional regularization; however, different renormalizaton and factorization prescriptions were applied [4–10]. The occurring differences are clarified in detail in Refs. [8,9]. Indeed, if one takes into account the errors, which are pointed out in Ref. [9], the results given in Refs. [5,9] can be obtained from those in Refs. [4,6–8,10]. The latter results are based on the common renormalization operation and the prescription also used in the calculation of the evolution kernel (a detailed discussion about these items is given in Ref. [8]):

$$\begin{aligned}
T^{(1)} &= C_F T^F(x,y,Q,\mu_F) + \beta_0 T^\beta(x,y,Q,\mu_R) + (C_F - C_A/2)\, T^{FA}(x,y), \\
T^F &= [3+\ln(xy)]\ln\left(\frac{Q^2}{\mu_F^2}\right) + \frac{1}{2}\ln^2(xy) + 3\ln(xy) - \frac{\ln x}{2(1-x)} - \frac{\ln y}{2(1-y)} - \frac{14}{3}, \\
T^\beta &= -\frac{1}{2}\ln\left(\frac{Q^2}{\mu_R^2}\right) - \frac{1}{2}\ln(xy) + \frac{5}{6},
\end{aligned} \tag{48}$$

$$
\begin{aligned}
T^{FA} &= \mathrm{Li}_2(1-x) - \mathrm{Li}_2(x) + \ln(1-x)\ln\left(\frac{y}{1-y}\right) - \frac{5}{3} \\
&+\frac{1}{(x-y)^2}\Bigg((x+y-2xy)\ln(1-x) + 2xy\ln(x) + \frac{(1-x)x^2+(1-y)y^2}{x-y} \\
&\times\left[\ln(1-x)\ln(y) - \mathrm{Li}_2(1-x) + \mathrm{Li}_2(x)\right]\Bigg) + \{x \leftrightarrow y\}.
\end{aligned}
$$

The appearing $\ln x$ and $\ln y$ terms are responsible for large contributions arising from the end-point region. Note that the singular end-point behaviour of $T^{FA}(x,y)$ is $-\ln(xy)$, which is due to the color structure suppressed by $1/N_c^2 = 1/9$. The appearing poles in $(x-y)$ are actually cancelled by zeros, implying that the corresponding term behaves smoothly for $x \to y$ (see Appendix A).

In contrast to the photon-to-pion transition form factor we do not have conformal predictions for the electromagnetic form factor that would enable us to predict higher loop corrections without explicit calculation. However, to analyse the NLO correction and to compare them in different schemes it is possible to employ the conformal spin expansion:

$$
\begin{aligned}
F_\pi(Q^2) &= \frac{2\pi f_\pi^2 C_F \alpha_s(\mu_R)}{3Q^2} {\sum_{n,m=0}^{\infty}}' \frac{2(3+2m)}{(m+1)(m+2)} \frac{2(3+2n)}{(n+1)(n+2)} \qquad (49)\\
&\times \langle \pi(P)|O_{mm}(\mu_F)|0\rangle^{red}\left[1+\frac{\alpha_s(\mu_R)}{2\pi}T^{(1)}_{mn}(Q,\mu_F,\mu_R)+\cdots\right]\langle 0|O_{nn}(\mu_F)|\pi(P)\rangle^{red},
\end{aligned}
$$

where the relative NLO correction is given by the moments:

$$
T^{(1)}_{mn} = 4\int_0^1 dx \int_0^1 dy\, (1-x) C_m^{\frac{3}{2}}(2x-1) T^{(1)}(x,y)(1-y) C_n^{\frac{3}{2}}(2y-1). \qquad (50)
$$

Using the shorthand notation

$$
\left\langle \frac{f(x)}{x}\right\rangle_m^{\mathrm{rel}} = \frac{\left\langle \frac{f(x)}{x}\right\rangle_m}{\left\langle \frac{1}{x}\right\rangle_m}, \quad \left\langle \frac{f(x)}{x}\right\rangle_m = \int_0^1 dx\, \frac{f(x)}{x}\frac{x(1-x)}{N_m} C_m^{\frac{3}{2}}(2x-1), \qquad (51)
$$

the result reads:

$$
\begin{aligned}
T^{(1)}_{mn} &= C_F T^F_{mn} + \beta_0 T^\beta_{mn} + (C_F - C_A/2)\, T^{FA}_{mn}, \qquad (52)\\
T^F_{mn} &= \left[3 + \left\langle \frac{\ln x}{x}\right\rangle_m^{\mathrm{rel}} + \left\langle \frac{\ln x}{x}\right\rangle_n^{\mathrm{rel}}\right] \ln\left(\frac{Q^2}{\mu_F^2}\right) + \frac{1}{2}\left\langle \frac{\ln^2 x}{x}\right\rangle_m^{\mathrm{rel}} + \frac{1}{2}\left\langle \frac{\ln^2 x}{x}\right\rangle_n^{\mathrm{rel}} + \left\langle \frac{\ln x}{x}\right\rangle_m^{\mathrm{rel}} \\
&\times \left\langle \frac{\ln x}{x}\right\rangle_n^{\mathrm{rel}} + \frac{5}{2}\left\langle \frac{\ln x}{x}\right\rangle_m^{\mathrm{rel}} + \frac{5}{2}\left\langle \frac{\ln x}{x}\right\rangle_n^{\mathrm{rel}} - \frac{14}{3} - \frac{1}{2}\left\langle \frac{\ln(1-x)}{x}\right\rangle_m^{\mathrm{rel}} - \frac{1}{2}\left\langle \frac{\ln(1-x)}{x}\right\rangle_n^{\mathrm{rel}}, \qquad (53)\\
T^\beta_{mn} &= -\frac{1}{2}\ln\left(\frac{Q^2}{\mu_R^2}\right) - \frac{1}{2}\left\langle \frac{\ln x}{x}\right\rangle_m^{\mathrm{rel}} - \frac{1}{2}\left\langle \frac{\ln x}{x}\right\rangle_n^{\mathrm{rel}} + \frac{5}{6}, \qquad (54)\\
T^{FA}_{mn} &= -\left\langle \frac{\ln x}{x}\right\rangle_m^{\mathrm{rel}}\left(1 - \left\langle \frac{\ln(1-x)+x}{x^2}\right\rangle_n^{\mathrm{rel}}\right) - \left\langle \frac{\ln x}{x}\right\rangle_n^{\mathrm{rel}}\left(1 - \left\langle \frac{\ln(1-x)+x}{x^2}\right\rangle_m^{\mathrm{rel}}\right) \\
&+\frac{\pi^2}{3} - \frac{7}{3} + \Delta T^{FA}_m + \Delta T^{FA}_n + \Delta T^{FA}_{mn}, \qquad (55)
\end{aligned}
$$

where ΔT_m^{FA} and ΔT_{mn}^{FA} are the (relative) conformal moments of the functions $\Delta T^{FA}(x)$ and $\Delta T^{FA}(x,y)$ defined in Eq. (A6). The singular terms in Eq. (48), given by $\ln^i(x)/x$ for $i=0,1,2$, provide a $\ln^i(m)$-behaviour and are analytically calculated in Appendix B. The remaining regular part provides ΔT_m^{FA}, ΔT_m^{FA} and ΔT_{mn}^{FA}, which are power and $1/N_c^2$ suppressed. Therefore, for all DA this part will become negligibly small for growing m and n and thus it is sufficient to calculate only the first few moments. The numerical results are listed in Appendix B, too.

For the chosen set of DA's it turns out that T^F gives a moderate negative contribution, while T^β contains a large positive contribution. There are different suggestions to optimize both the factorization and renormalization scale setting. Here we will skip the factorization scale setting problem (detailed discussions are given for instance in Refs. [5,10,68]) and deal only with the renormalization scale setting prescription. This contribution arises only from the renormalization of the coupling and is proportional to β_0. Thus, it can be absorbed in a quite natural way by the BLM prescription into the scale of the coupling [67]:

$$\mu_R \to Q_{mn}^* = Q \exp\left\{-\frac{1}{2}\left\langle\frac{\ln x}{x}\right\rangle_m^{\rm rel} - \frac{1}{2}\left\langle\frac{\ln x}{x}\right\rangle_n^{\rm rel} + \frac{5}{6}\right\}, \tag{56}$$

with $T_{mn}^\beta(Q, Q_{mn})$ vanishing. For the "natural" scale setting prescription $\mu_F = Q$, the form factor can be written after summation as

$$\begin{aligned} F_\pi(Q) &= \frac{2\pi C_F f_\pi^2 \alpha_s(Q^*)}{3Q^2}\left\{1 + \frac{\alpha_s}{2\pi} r^{(1)}(Q,Q) + O\left(\alpha_s^2\right)\right\} I(Q,Q_0)^2, \\ &= \frac{\alpha_s(Q^*)}{Q^2}\left\{1 + \frac{\alpha_s}{2\pi} r^{(1)}(Q,Q) + O\left(\alpha_s^2\right)\right\} C_\pi(Q,Q_0), \end{aligned} \tag{57}$$

where $C_\pi(Q,Q_0) = 2\pi C_F f_\pi^2 I(Q,Q_0)^2/3$ contains all evolution effects, $Q^* = Qe^{-\Delta}$ and the I integral is defined in Eq. (29). To obtain the NLO corrections as scheme independent as possible, one should take into account the higher order corrections due to the evolution in the following manner:

$$C_\pi(Q,Q_0) = C_\pi^{\rm LO}(Q,Q_0)\left(1 + \frac{\alpha_s}{2\pi}\cdots + O(\alpha_s^2)\right) \tag{58}$$

and truncate then the series in Eq. (57) in the first order in α_s. Fortunately, for a reference momentum square of $Q \geq 2$ GeV the size of the NLO contributions arising from the evolution allows us to use Eq. (57) in practice. In Table III we give the results for $C_\pi(Q_0,Q_0)$, $r^{(1)}$ and Δ at $Q^2 = 2$ GeV^2 for the chosen set of DAs and for the three considered schemes. To carry out the calculation more easily, the scale setting has been done after summation.

Finally, we discuss the phenomenological consequences of the perturbative NLO corrections. Due to the large value of Δ the scale Q^* appearing in the coupling will be very small. For instance, at $Q = \sqrt{2}$ GeV we find for the asymptotic DA $Q^* \approx 0.14$ GeV and for the two-hump DA $Q^* \approx 0.08$ GeV in the $\overline{\rm MS}$ and $\rm CS_I$ schemes. At this low scale the perturbative treatment of the coupling is not valid anymore. However, it was argued that the coupling is frozen at low momentum transfer. A simple parametrization of such a frozen coupling arises from the idea that the gluon propagator has an effective mass m_g that is induced by non-perturbative effects [69–71]:

DA	C_π	$\overline{\rm MS}$ $r^{(1)}$	$\overline{\rm MS}$ Δ	$\mathrm{CS_I}$ $r^{(1)}$	$\mathrm{CS_I}$ Δ	$\mathrm{CS_{II}}$ $r^{(1)}$	$\mathrm{CS_{II}}$ Δ
φ^{10}	0.211	-8.1	1.60	-5.7	1.60	-5.7	1.42
$\varphi^{\rm as}$	0.431	-7.8	2.33	-5.2	2.33	-5.2	2.00
$\varphi^{1/2}$	0.767	-1.22	3.22	0.82	3.22	0.82	2.62
$\varphi^{\rm CZ}$	0.848	-5.9	2.93	-3.1	2.93	-3.1	2.46
$\varphi^{1/4}$	1.725	34.4	5.10	29.8	5.10	29.8	3.8

TABLE III. LO predictions and NLO corrections for the hard-scattering amplitude of the electromagnetic pion form factor for different DAs, where the evolution is neglected and the factorization scale is set to Q. The renormalization scale is fixed by the BLM procedure.

$$\alpha_s^{\rm fro}(Q) = \frac{4\pi}{\beta_0 \ln\left(\frac{Q^2+4m_g^2}{\Lambda^2}\right)}. \tag{59}$$

In the following we assume that $\alpha_s^{\rm fro}$ is about 0.5 at very low momentum transfer [72,67]. Employing Eq. (57) we obtain from Table III the NLO prediction for the electromagnetic form factor. In the conformal scheme $\mathrm{CS_I}$ the prediction is $Q^2F_\pi \approx 0.12$ for the asymptotic DA and $Q^2F_\pi \approx 0.32$ for the two-hump DA. These results are compatible with the LO predictions shown in Fig. 2.

VI. CONCLUSIONS

In this paper we have studied the scheme dependence of the photon-to-pion transition form factor and the electromagnetic pion form factor in NLO for two different schemes, namely, the popular $\overline{\rm MS}$ scheme and the so-called conformal scheme. In the case of two-particle distribution amplitudes, the latter one is uniquely defined in the conformal limit of the theory by the requirement of conformal covariance. Obviously, beyond this limit we have to deal with an ambiguity that is proportional to the β function. Such terms can be naturally included in the scale of the coupling by the BLM scale setting prescription. However, as we saw in Subsection III B in the case of exclusive processes this prescription is not sufficient to restore the conformal covariance of the perturbative prediction with respect to the original representation. We may expect that for so-called reducible but non-decomposable representations conformal covariance holds true.

Instead of dealing with the usual convolution of hard scattering part and DAs we exploit the possibility to work directly with the conformal moments, so that the QCD prediction for exclusive processes is given as a sum about such moments. The advantage of such representation is that the evolution of the moments is easy to handle and the back transformation into the x-space by infinite sums over Gegenbauer polynomials can be avoided. Although such sums can be calculated with an appropriate accuracy in a straightforward way, the numerical cancellation of the oscillations requires some care. For the method, which we used here, the exclusive predictions by themselfs are given by infinite sums that do not suffer

under oscillations and so they can be calculated without difficulties by taking into account a sufficient large number of terms. Especially, for the photon-to-pion transition form factor this method allows us to immediately use information available from DIS to give a prediction beyond the NLO.

The NLO corrections in the conformal scheme $\mathrm{CS_I}$ are in general smaller than in the $\overline{\mathrm{MS}}$ scheme. This observation supports the general argument that one should choose a scheme in which the underlying symmetries of the theory are preserved in the maximal possible manner. In the case of conformal symmetry we cannot fix the scheme uniquely, since the renormalization of the coupling causes a conformal anomaly proportional to the β function. As we demonstrated this remaining freedom can provide us huge differences for the NLO corrections. In the case of the photon-to-pion transition form factor we saw that due to this freedom the differences between the prediction of end-point narrow and concentrated DAs are starting to be compatible with the data. However, one has to be very careful with some conclusions. Since we did not transform the DAs, the real question arising here is: To which scheme belong the non-perturbative input DAs? It seems to us that this problem should be considered with more attention.

It is also desirable to go beyond the NLO to get more insight in the perturbative corrections to exclusive processes. At least for the photon-to-pion transition form factor the conformal techniques can be extended for the calculation of the NNLO corrections to the coefficient function in the full theory.

APPENDIX A: DECOMPOSITION OF T^{FA}

In this appendix we extract from $T^{FA}(x,y)$, defined in Eq. (48), that part which vanishes in the limit $x, y \to 0$. Moreover, we derive for these terms an expansion which can be conveniently used in the convolution of the hard-scattering amplitude with the DA's. The most complicated looking term in $T^{FA}(x,y)$ is proportional to $1/(x-y)^2$:

$$t^{FA} = \frac{1}{(x-y)^2}\Bigg((x+y-2xy)\ln(1-x) + 2xy\ln(x) + \frac{(1-x)x^2+(1-y)y^2}{x-y} \\ \times \left[\ln(1-x)\ln(y) - \mathrm{Li}_2(1-x) + \mathrm{Li}_2(x)\right]\Bigg) + \{x \leftrightarrow y\}. \tag{A1}$$

Using the simple algebraic decomposition

$$\frac{(1-x)x^2+(1-y)y^2}{(x-y)^3} = \frac{1-x-y}{x-y} + \frac{(2-x-y)xy}{(x-y)^3} \tag{A2}$$

and the equality

$$\mathrm{Li}_2(1-x) = -\mathrm{Li}_2(x) - \ln(1-x)\ln x + \frac{\pi^2}{6}, \tag{A3}$$

we immediately obtain for t^{FA}:

$$
\begin{aligned}
t^{FA} &= \frac{1}{x-y}\left[\{1+(1-x-y)\ln(xy)\}\ln\left(\frac{1-x}{1-y}\right)+2(1-x-y)\left\{\mathrm{Li}_2(x)-\mathrm{Li}_2(y)\right\}\right] \\
&+\frac{\ln(xy)}{(x-y)^2}\left[2xy+\frac{(2-x-y)xy}{x-y}\ln\left(\frac{1-x}{1-y}\right)\right]+\frac{1}{(x-y)^2}\Bigg[(x+y-2xy) \\
&\times\ln\left[(1-x)(1-y)\right]+\frac{2(2-x-y)xy}{x-y}\left[\mathrm{Li}_2(x)-\mathrm{Li}_2(y)\right]-(x-y)\ln\left(\frac{1-x}{1-y}\right)\Bigg].
\end{aligned} \tag{A4}
$$

To extract the desired part, one should take into account the following set of algebraic identities:

$$
\begin{aligned}
\frac{f(x)-f(y)}{x-y} &= -1+\frac{x+f(x)}{x}+\frac{y+f(y)}{y}+\frac{y^2f(x)-(x-y)xy-x^2f(y)}{(x-y)xy}, \\
\frac{(1-x-y)[f(x)-f(y)]}{x-y} &= \mp 1+\frac{\pm x+(1-x)f(x)}{x}+\frac{\pm y+(1-y)f(y)}{y} \\
&+\frac{(1-2x)y^2f(x)\mp(x-y)xy-(1-2y)x^2f(y)}{(x-y)xy},
\end{aligned} \tag{A5}
$$

where in the last equation the upper [lower] sign is used for $f(x)=\ln(1-x)\,[\mathrm{Li}_2(x)]$. Note that the third term on the RHS vanishes for $x,y\to 0$. Taking also into account the remaining part of $T^{FA}(x,y)$, we finally obtain after some simple aglebra the desired representation:

$$
\begin{aligned}
T^{FA}(x,y) &= -\ln(x)\left[1-\frac{\ln(1-y)+y}{y}\right]-\ln(y)\left[1-\frac{\ln(1-x)+x}{x}\right]+\frac{\pi^2-7}{3} \\
&+\Delta T^{FA}(x)+\Delta T^{FA}(y)+\Delta T^{FA}(x,y), \\
\Delta T^{FA}(x) &= \frac{\ln(1-x)+x}{x}+\frac{\ln(x)\left[(1-2x)\ln(1-x)+x\right]}{x}+2\frac{(1-2x)\mathrm{Li}_2(x)-x}{x}, \\
\Delta T^{FA}(x,y) &= -2\ln(1-x)\ln(1-y)+\theta^{FA}(x,y), \\
\theta^{FA}(x,y) &= \frac{\ln(xy)}{(x-y)xy}\left[y^2(1-2x)\ln(1-x)-xy(x-y)-x^2(1-2y)\ln(1-y)\right] \\
&+\frac{1}{(x-y)xy}\Bigg[y^2\left\{2(1-2x)\mathrm{Li}_2(x)+\ln(1-x)\right\}+xy(x-y) \\
&\qquad -x^2\left\{2(1-2y)\mathrm{Li}_2(y)+\ln(1-y)\right\}-xy\ln\left(\frac{1-x}{1-y}\right)\Bigg] \\
&+\frac{\ln(xy)}{(x-y)^2}\left[2xy+\frac{(2-x-y)xy}{x-y}\ln\left(\frac{1-x}{1-y}\right)\right]+\frac{1}{(x-y)^2} \\
&\times\left[(x+y-2xy)\ln\left[(1-x)(1-y)\right]+\frac{2(2-x-y)xy}{x-y}\left[\mathrm{Li}_2(x)-\mathrm{Li}_2(y)\right]\right],
\end{aligned} \tag{A6}
$$

where $\Delta T^{FA}(x)$ and $\Delta T^{FA}(x,y)$ vanish for $x,y\to 0$. Especially, for $\Delta T^{FA}(x,y)$ this can be seen by a power expansion of $\theta^{FA}(x,y)$, namely,

$$
\theta^{FA}=xy\sum_{i=0}^{\infty}\sum_{j=0}^{i}\left(\frac{3-ij+j^2}{(2+i)(3+i)}\ln(xy)-\frac{2(1-i-ij+j^2)}{(2+i)^2}-\frac{3+3i+2ij-2j^2}{(3+i)^2}\right)x^jy^{i-j}. \tag{A7}
$$

It is now also obvious that $\theta^{FA}(x,y)$ is regular at $x=y$. In the calculation of the NLO corrections it is justified to approximate $\theta^{FA}(x,y)$ by the first few terms of this expansion.

APPENDIX B: CONFORMAL MOMENTS

Here we list the needed conformal expansions (with respect to the Gegenbauer polynomials) of terms appearing in the hard-scattering amplitude of the pion form factor to NLO. They are obtained by employing the representation

$$\frac{x(1-x)}{N_k} C_k^{\frac{3}{2}}(2x-1) = (-1)^k \frac{2(3+2k)}{(k+1)!} \frac{d^k}{dx^k} x^{k+1}(1-x)^{k+1} \tag{B1}$$

as well as the definition of the B-function

$$B(a,b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)} = \int_0^1 dx\, x^{a-1}(1-x)^{b-1}. \tag{B2}$$

$$\begin{aligned} \Big\langle \frac{\ln^i x}{x} \Big\rangle_k &= \frac{\partial^i}{\partial \epsilon^i} \int_0^1 dx\ x^{\epsilon-1} \frac{x(1-x)}{N_k} C_k^{\frac{3}{2}}(2x-1)_{|\epsilon=0} \\ &= \frac{\partial^i}{\partial \epsilon^i} \frac{2(3+2k)}{(k+1)!} \int_0^1 dx \left[\frac{d^k}{dx^k} x^{\epsilon-1} \right] x^{k+1}(1-x)^{k+1}_{|\epsilon=0} \\ &= (-1)^k \frac{\partial^i}{\partial \epsilon^i} \left[\frac{2(3+2k)\Gamma(1+\epsilon)\Gamma(k+1-\epsilon)}{\Gamma(1-\epsilon)\Gamma(k+3+\epsilon)} \right]_{|\epsilon=0} . \end{aligned} \tag{B3}$$

For $i=0$ we immediately find

$$\Big\langle \frac{1}{x} \Big\rangle_k = (-1)^k \frac{2(3+2k)}{(k+1)(k+2)}. \tag{B4}$$

Applying the definition of the digamma function $\psi(x) = \frac{d}{dx} \ln \Gamma(x)$ provides

$$\Big\langle \frac{\ln x}{x} \Big\rangle_k = (-1)^k \frac{2(3+2k)}{(k+1)(k+2)} \left[2\psi(1) - \psi(k+1) - \psi(k+3) \right], \tag{B5}$$

$$\begin{aligned} \Big\langle \frac{\ln^2 x}{x} \Big\rangle_k = (-1)^k \frac{2(3+2k)}{(k+1)(k+2)} \Big\{ &\left[2\psi(1) - \psi(k+1) - \psi(k+3) \right]^2 \\ &+\psi'(k+1) - \psi'(k+3) \Big\}. \end{aligned} \tag{B6}$$

For large k we find with $\psi(k+1) - \psi(1) = \ln(k+1) + O(1/k)$ the following asymptotic behaviour

$$\Big\langle \frac{\ln^i x}{x} \Big\rangle_k = \Big\langle \frac{1}{x} \Big\rangle_k \left\{ \left[2\ln(k+1) \right]^i + O(1/k) \right\}, \tag{B7}$$

which reflects the singular end-point behaviour for $x \to 0$.

Closed formulas for $\langle \frac{\ln(1-x)}{x} \rangle_k$ and $\langle \frac{\ln(1-x)+x}{x^2} \rangle_k$ can be derived by employing the expansion $\frac{\ln(1-x)}{x} = -\sum_{i=0}^{\infty} x^i/(i+1)$. After integration over x the summation can be carried out:

$$\left\langle \frac{\ln(1-x)}{x} \right\rangle_k = -\frac{2(3+2k)}{(k+2)!} \sum_{n=k}^{\infty} \frac{n!}{(n-k)!} B(n+1, k+3) = -\frac{2(3+2k)}{(k+1)^2(k+2)^2}. \tag{B8}$$

In an analogous way one finds:

$$\begin{aligned} \left\langle \frac{\ln(1-x)+x}{x^2} \right\rangle_k &= -\frac{2(3+2k)}{(k+1)!} \sum_{n=k}^{\infty} \frac{1}{n+2} \frac{n!}{(n-k)!} B(n+2, k+2) \\ &= -\frac{2(3+2k)}{(k+1)(k+2)^2(k+3)} {}_3F_2 \left(\left. \begin{matrix} k+2, 1, 2 \\ k+3, k+4 \end{matrix} \right| 1 \right). \end{aligned} \tag{B9}$$

Finally, the numerical results that are needed for the calculation of the pion form factor to NLO will be listed in Tables IV and V. Let us remark that the moments ΔT^{FA}_{mn} can be

m	0	2	4	6	8	10	12	14
$\left\langle \frac{\ln(1-x)+x}{x^2} \right\rangle^{\rm rel}_m$	-0.290	-0.073	-0.031	-0.017	-0.011	-0.007	-0.005	-0.004
ΔT^{FA}_m	-2.157	-0.226	-0.086	-0.045	-0.028	-0.019	-0.013	-0.010

TABLE IV. Numerical results for the first few moments of $\left\langle \frac{\ln(1-x)+x}{x^2} \right\rangle^{\rm rel}_m$ and ΔT^{FA}_m. The latter is the relative conformal moment of $\Delta T^{FA}(x)$, defined in Eq. A6, and appears in Eq. (55)

$n \backslash m$	0	2	4	6	8	10
0	-0.845	-0.129	-0.050	-0.026	-0.016	-0.010
2	-0.129	-0.020	-0.008	-0.005	-0.003	-0.002
4	-0.050	-0.008	-0.004	-0.002	-0.002	-0.001
6	-0.026	-0.005	-0.002	-0.002	-0.001	-0.001
8	-0.016	-0.003	-0.002	-0.001	-0.001	-0.001
10	-0.010	-0.002	-0.001	-0.001	-0.001	0.000

TABLE V. Numerical results for the first few relative conformal moment of $\Delta T^{FA}(x,y)$ defined in Eq. A6. They appear in Eq. (55).

easily calculated by employing the representation (A7).

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